



Rates of Change and Limits

- 1** Find the average rate of change of this function from this point over an interval of width h .

$$f(x) = \frac{1}{x}$$

$$f(2) = 0.5$$

$$f(2+h) = \frac{1}{2+h}$$

A	$\frac{-1}{0+2h}$	B	$\frac{-1}{2+h}$
C	$\frac{1}{4+2h}$	D	$\frac{-1}{4+2h}$

- 2** Find the average rate of change of this function from this x-value over an interval of width h .

$$f(x) = 3x^2$$

$$f(-1) = ?$$

$$f(-1+h) = ?$$

A	$-6 - 3h$
B	$-3 + 3h$
C	-6
D	$-6 + 3h$

- 3** Find the average rate of change of this function between the two points.

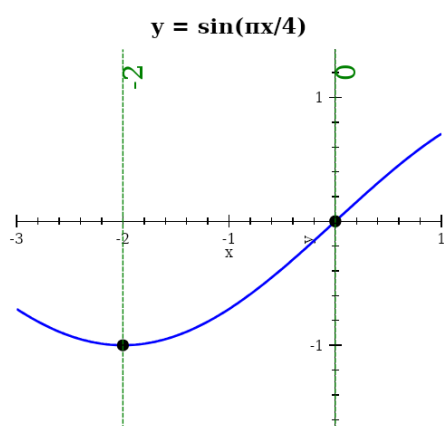
$$f(x) = 2x^3$$

$$f(-1) = -2$$

$$f(1) = 2$$

A	-2	B	4	C	2
D	0	E	0.5		

- 4** Find the average rate of change between the two highlighted x-values $x=0$ and $x=-2$.



A	0.5	B	2.2	C	2
D	-1	E	-0.5		

- 5** Find the average rate of change of this function between the two x-values.

$$f(x) = 3x - 1$$

$$f(0) = ?$$

$$f(-2) = ?$$

A	-3	B	-6	C	0.3
D	0	E	3		

- 6** Evaluate the limit.

$$\lim_{x \rightarrow -2} (-x^2 + 2x - 1)$$

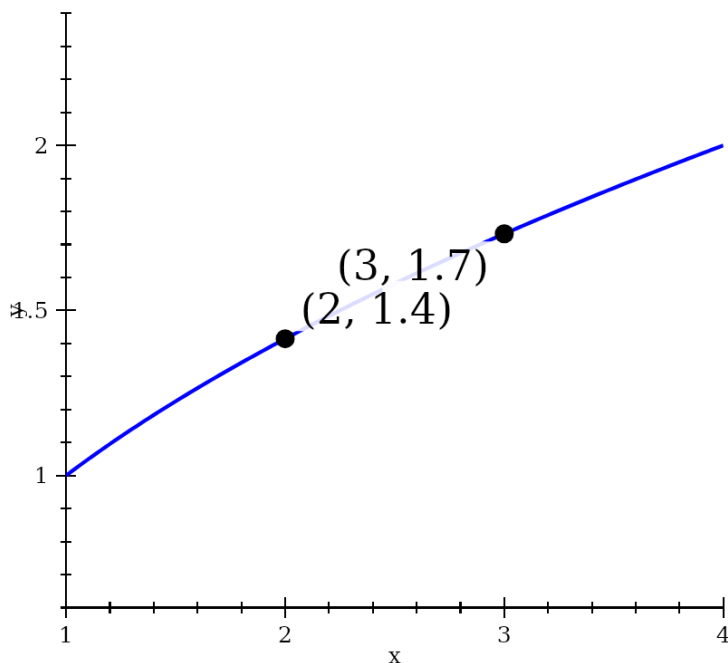
A	-7	B	-10
C	-9	D	-8



Rates of Change and Limits

- 7 Find the average rate of change between the two marked points (2, 1.4) and (3, 1.7).

$$y = \sqrt{x}$$



A

-0.3

B

2.3

C

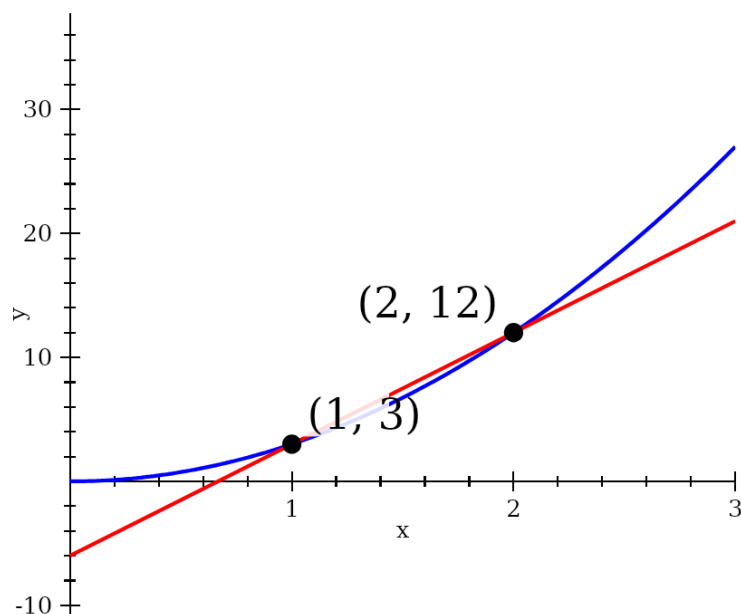
3.1

D

0.3

- 8 Find the average rate of change shown by the secant line between the two marked points (1, 3) and (2, 12).

$$y = 3x^2$$



A

0.1

B

-9

C

9.5

D

9



Rates of Change and Limits

9 Use the table to find the value $f(x)$ approaches as x approaches -1 from the left.

x	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
f(x)	-1.1	-1.01	-1.001	-1	-1.002	-1.02	-1.2

A

$$\lim_{x \rightarrow -1^-} f(x) = 0$$

B

$$\lim_{x \rightarrow -1^-} f(x) = -2$$

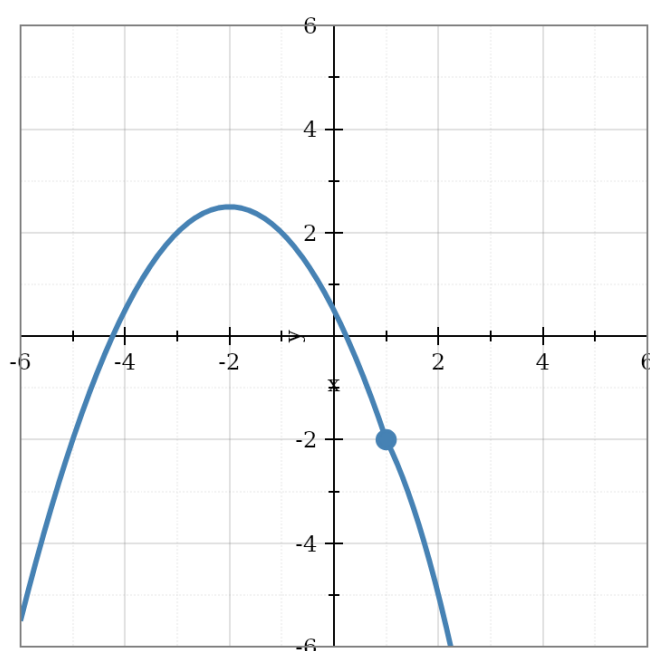
C

$$\lim_{x \rightarrow -1^-} f(x) = -1$$

D

$$\lim_{x \rightarrow -1^-} f(x) = 1$$

10 Use the graph to evaluate the one-sided limit as x approaches 1 from the right.



A

$$\lim_{x \rightarrow 1^+} f(x) = -3$$

B

$$\lim_{x \rightarrow 1^+} f(x) = -2$$

C

$$\lim_{x \rightarrow 1^+} f(x) = -1$$

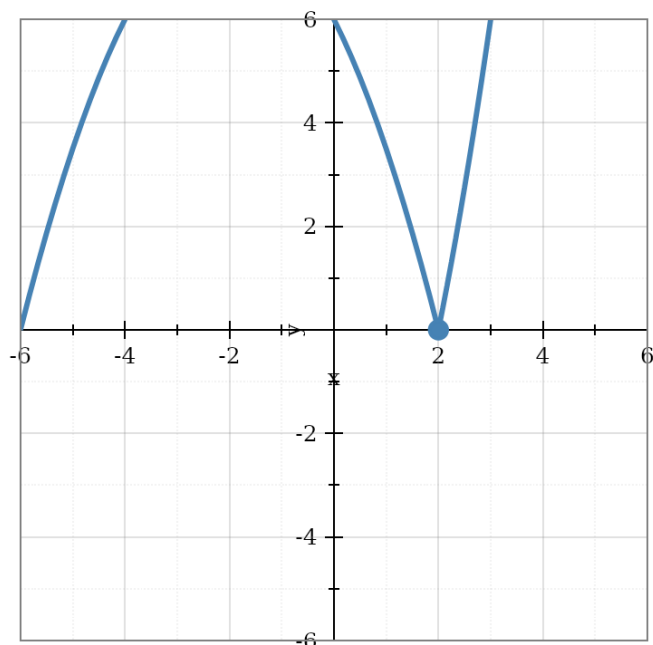
D

$$\lim_{x \rightarrow 1^+} f(x) = 0$$



Rates of Change and Limits

11 Use the graph to find the value $f(x)$ approaches as x approaches 2 from the left.



A

$$\lim_{x \rightarrow 2^-} f(x) = 0$$

B

$$\lim_{x \rightarrow 2^-} f(x) = 1$$

C

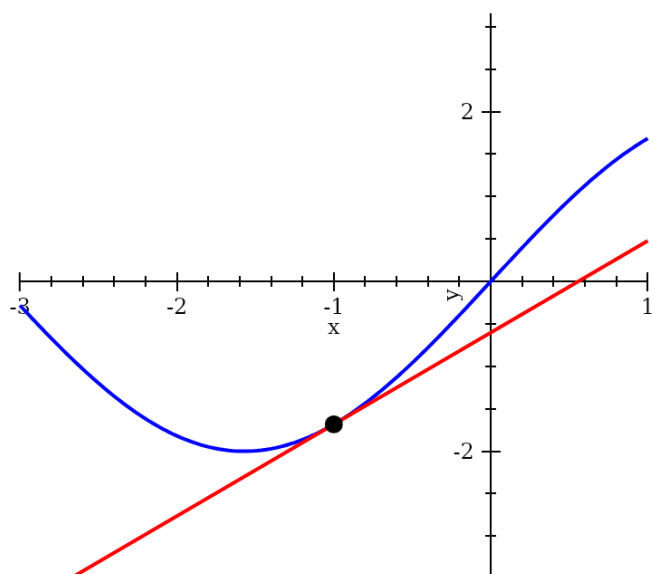
$$\lim_{x \rightarrow 2^-} f(x) = 2$$

D

$$\lim_{x \rightarrow 2^-} f(x) = -1$$

12 Estimate the instantaneous rate of change (the slope of the tangent line) at the marked point. Note: the y-axis may not be the same scale as the x-axis.

$$y = 2\sin(x)$$



A

about 0.5

B

about 1.1

C

about -1

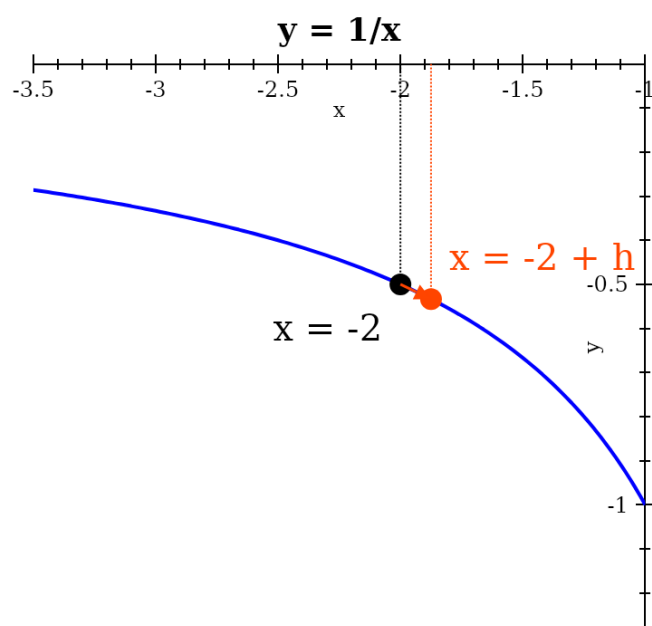
D

about -0.5



Rates of Change and Limits

- 13** Estimate the instantaneous rate of change at the marked point by adding a second point very near to it. Note: the y-axis may not be the same scale as the x-axis.



A

about -0.3

B

about 0.5

C

about 1.5

D

about 1

- 14** Evaluate the limit.

$$\lim_{x \rightarrow -1} \frac{-2x + 4}{-2x + 2}$$

A $-\frac{3}{2}$

B $\frac{3}{2}$

C $\frac{2}{3}$

D $\frac{7}{4}$

- 15** Multiply by the conjugate, manipulate and cancel, then evaluate the limit.

$$\lim_{x \rightarrow 3} \frac{x - 3}{\sqrt{x + 13} - 4}$$

A 8

B 7

C 9

D 10

- 16** Multiply by the conjugate, cancel, then evaluate the limit.

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$$

A $\frac{1}{4}$

B 4

C $-\frac{1}{4}$

D $\frac{1}{2}$

- 17** Factor and cancel, then evaluate the limit.

$$\lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1}$$

A 2

B 1

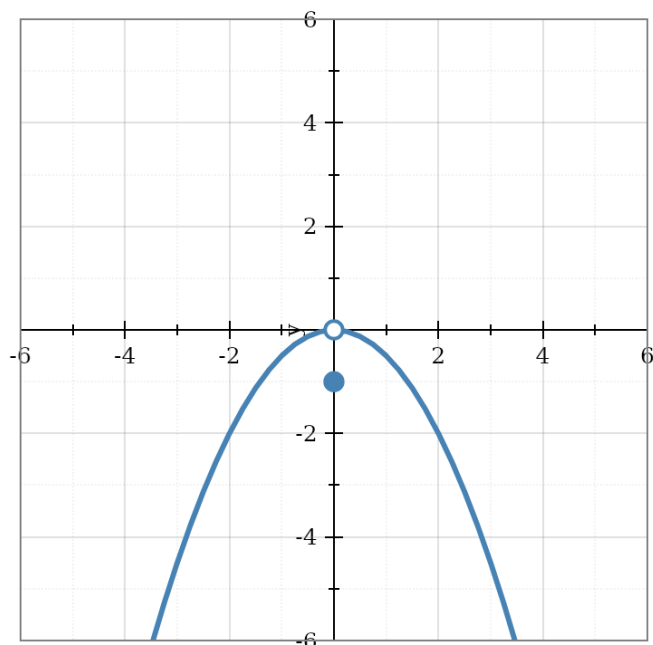
C 3

D 0



Rates of Change and Limits

18 Use the graph to evaluate $\lim_{x \rightarrow 0} f(x)$.



A

$$\lim_{x \rightarrow 0} f(x) = 1$$

B

$$\lim_{x \rightarrow 0} f(x) = -1$$

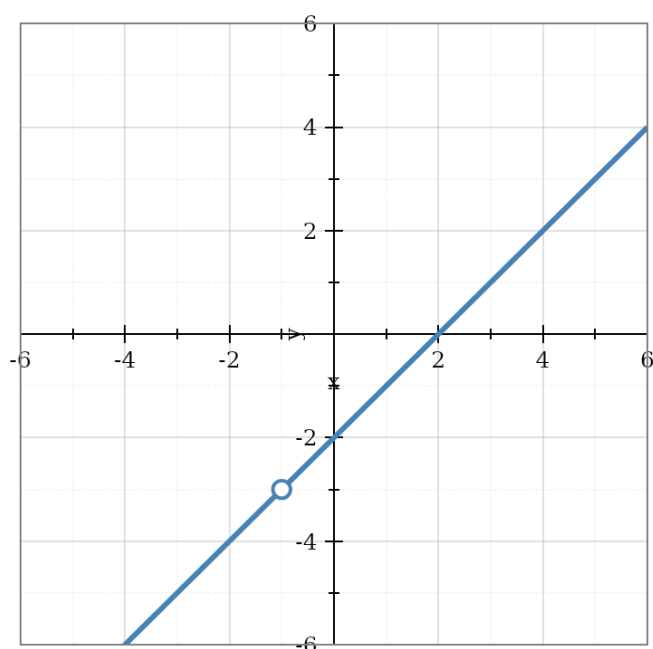
C

$$\lim_{x \rightarrow 0} f(x) = 2$$

D

$$\lim_{x \rightarrow 0} f(x) = 0$$

19 Use the graph to evaluate $\lim_{x \rightarrow -1} f(x)$.



A

$$\lim_{x \rightarrow -1} f(x) = -1$$

B

$$\lim_{x \rightarrow -1} f(x) = -2$$

C

$$\lim_{x \rightarrow -1} f(x) = -3$$

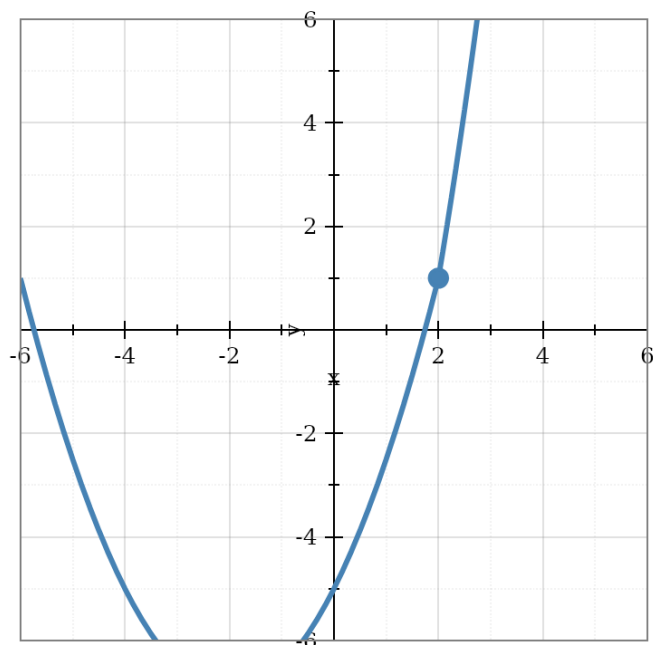
D

$$\lim_{x \rightarrow -1} f(x) = -4$$



Rates of Change and Limits

20 Use the graph to evaluate $\lim_{x \rightarrow 2} f(x)$ (the two-sided limit).



A

$$\lim_{x \rightarrow 2} f(x) = 0$$

B

$$\lim_{x \rightarrow 2} f(x) = 1$$

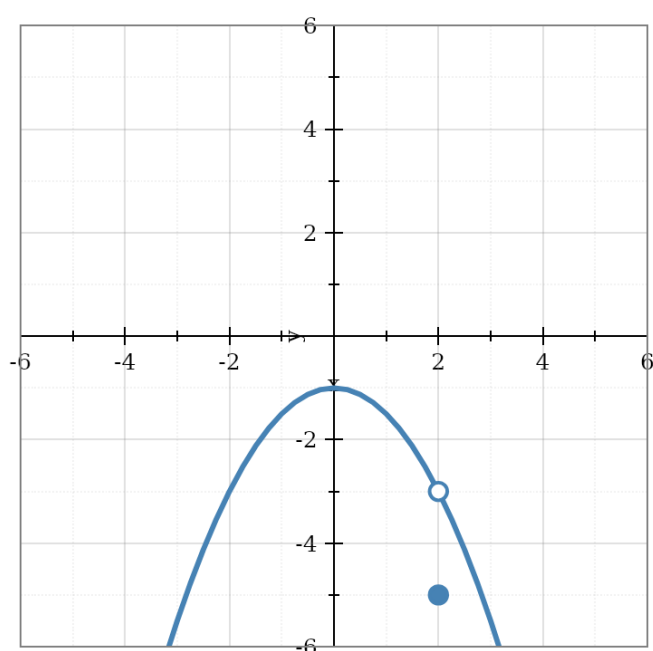
C

$$\lim_{x \rightarrow 2} f(x) = 3$$

D

$$\lim_{x \rightarrow 2} f(x) = 2$$

21 Use the graph to find the value $f(x)$ approaches as x approaches 2, and the value of $f(2)$.



A

$$\lim_{x \rightarrow 2} f(x) = -3, f(2) = -3$$

B

$$\lim_{x \rightarrow 2} f(x) = -5, f(2) = -5$$

C

$$\lim_{x \rightarrow 2} f(x) = -3, f(2) = -5$$

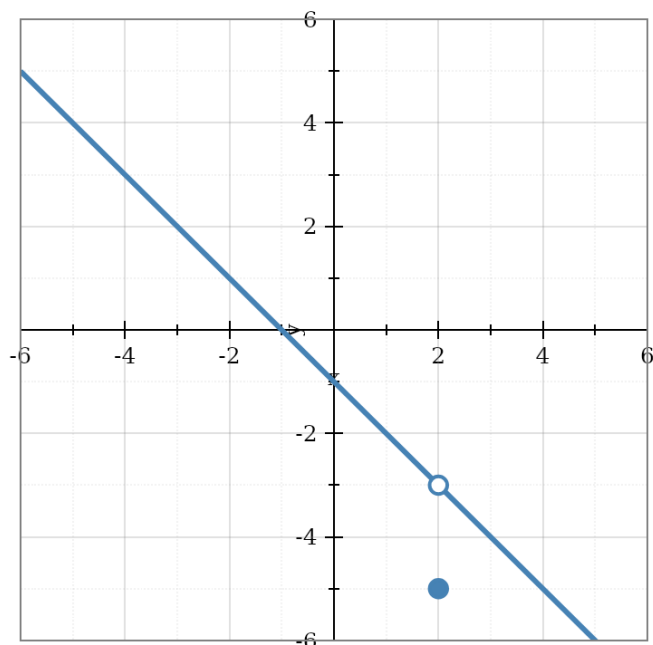
D

$$\lim_{x \rightarrow 2} f(x) = -5, f(2) = -3$$



Rates of Change and Limits

22 Use the graph to find the value $f(x)$ approaches as x gets closer to 2.



A

$$\lim_{x \rightarrow 2} f(x) = -2$$

B

$$\lim_{x \rightarrow 2} f(x) = -1$$

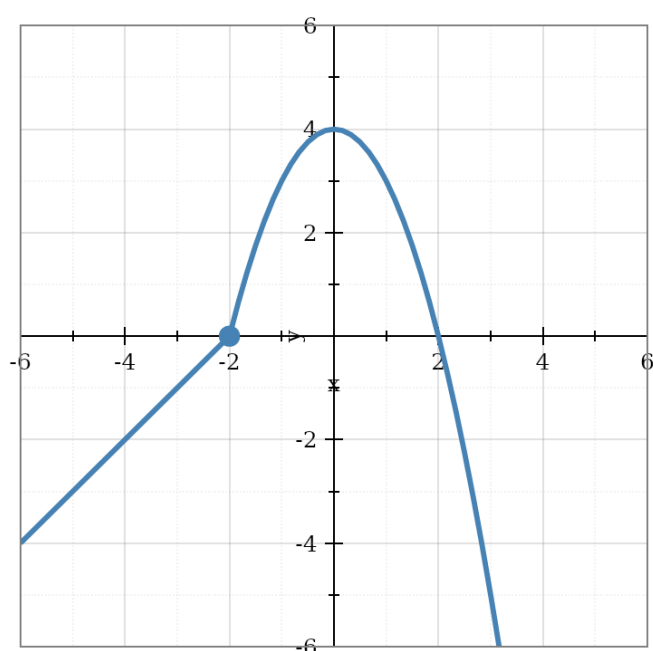
C

$$\lim_{x \rightarrow 2} f(x) = -3$$

D

$$\lim_{x \rightarrow 2} f(x) = -4$$

23 Use the graph to find the value $f(x)$ approaches as x approaches -2 from both sides.



A

$$\lim_{x \rightarrow -2} f(x) = 2$$

B

$$\lim_{x \rightarrow -2} f(x) = 1$$

C

$$\lim_{x \rightarrow -2} f(x) = 0$$

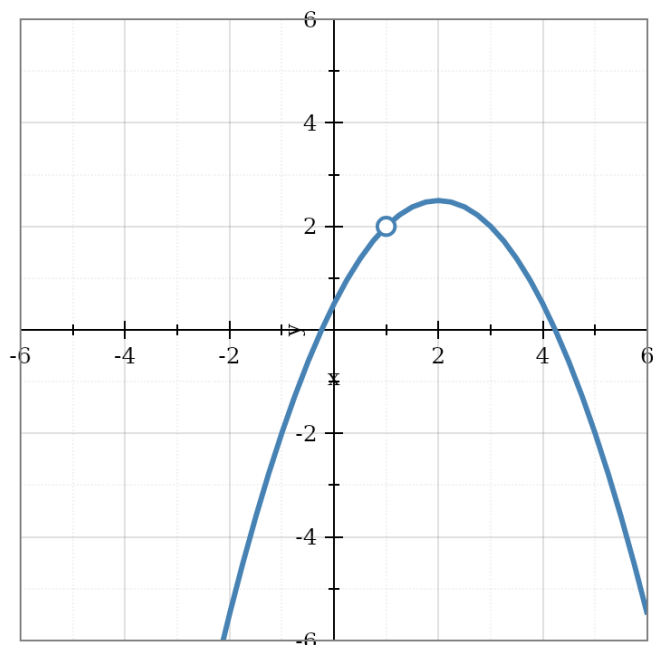
D

$$\lim_{x \rightarrow -2} f(x) = -1$$



Rates of Change and Limits

24 Use the graph to find the value $f(x)$ approaches as x gets closer to 1.



A

$$\lim_{x \rightarrow 1} f(x) = 1$$

B

$$\lim_{x \rightarrow 1} f(x) = 4$$

C

$$\lim_{x \rightarrow 1} f(x) = 3$$

D

$$\lim_{x \rightarrow 1} f(x) = 2$$

25 Use the table to evaluate $\lim_{x \rightarrow -1} f(x)$ (the two-sided limit).

x	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
$f(x)$	-3.1	-3.01	-3.001	-3	1.001	1.01	1.1

A

$$\lim_{x \rightarrow -1} f(x) = -3$$

B

$$\lim_{x \rightarrow -1} f(x) = -2$$

C

$$\lim_{x \rightarrow -1} f(x) \text{ does not exist}$$

D

$$\lim_{x \rightarrow -1} f(x) = 1$$



Rates of Change and Limits

26 Use the table to find the value $f(x)$ approaches as x gets closer to -2 .

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
f(x)	-2.9	-2.99	-2.999	undefined	-3.001	-3.01	-3.1

A

$$\lim_{x \rightarrow -2} f(x) = -3$$

B

$$\lim_{x \rightarrow -2} f(x) = -4$$

C

$$\lim_{x \rightarrow -2} f(x) = -2$$

D

$$\lim_{x \rightarrow -2} f(x) = -1$$

27 Use the table to evaluate $\lim_{x \rightarrow -2} f(x)$.

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
f(x)	-3.1	-3.01	-3.001	0	-2.999	-2.99	-2.9

A

$$\lim_{x \rightarrow -2} f(x) = -4$$

B

$$\lim_{x \rightarrow -2} f(x) = -2$$

C

$$\lim_{x \rightarrow -2} f(x) = -3$$

D

$$\lim_{x \rightarrow -2} f(x) = -1$$



Rates of Change and Limits

28 Use the table to evaluate $\lim_{x \rightarrow 2} f(x)$.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	1.9	1.99	1.999	undefined	2.001	2.01	2.1

A

$$\lim_{x \rightarrow 2} f(x) = 1$$

B

$$\lim_{x \rightarrow 2} f(x) = 2$$

C

$$\lim_{x \rightarrow 2} f(x) = 3$$

D

$$\lim_{x \rightarrow 2} f(x) = 4$$

29 Use the table to find the value $f(x)$ approaches as x approaches -2 , and the value of $f(-2)$.

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
f(x)	-0.8	-0.98	-0.998	2	-1.002	-1.02	-1.2

A

$$\lim_{x \rightarrow -2} f(x) = 2, f(-2) = -1$$

B

$$\lim_{x \rightarrow -2} f(x) = -1, f(-2) = -1$$

C

$$\lim_{x \rightarrow -2} f(x) = 2, f(-2) = 2$$

D

$$\lim_{x \rightarrow -2} f(x) = -1, f(-2) = 2$$



Rates of Change and Limits

30 Use the table to find the value $f(x)$ approaches as x gets closer to -1 .

x	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9
f(x)	-3.1	-3.01	-3.001	-2	-2.999	-2.99	-2.9

A

$$\lim_{x \rightarrow -1} f(x) = -2$$

B

$$\lim_{x \rightarrow -1} f(x) = -4$$

C

$$\lim_{x \rightarrow -1} f(x) = -1$$

D

$$\lim_{x \rightarrow -1} f(x) = -3$$

31 Use the table to find $\lim_{x \rightarrow -2} f(x)$ and the value $f(-2)$.

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
f(x)	-2.9	-2.99	-2.999	-4	-3.001	-3.01	-3.1

A

$$\lim_{x \rightarrow -2} f(x) = -4, f(-2) = -4$$

B

$$\lim_{x \rightarrow -2} f(x) = -3, f(-2) = -4$$

C

$$\lim_{x \rightarrow -2} f(x) = -4, f(-2) = -3$$

D

$$\lim_{x \rightarrow -2} f(x) = -3, f(-2) = -3$$



Rates of Change and Limits

32 Use the table to find the value $f(x)$ approaches as x approaches 1 from both sides.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
f(x)	-2.1	-2.01	-2.001	-2	-3.002	-3.02	-3.2

A

$$\lim_{x \rightarrow 1} f(x) = -3$$

B

$$\lim_{x \rightarrow 1} f(x) = -1$$

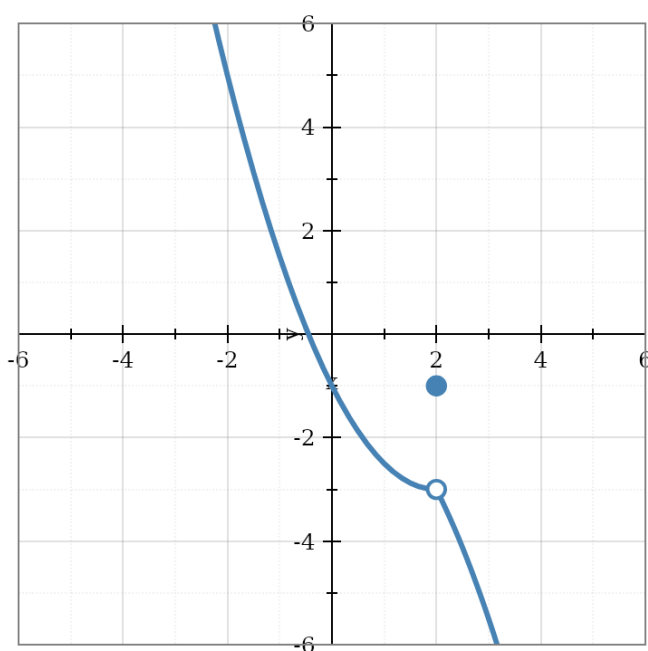
C

$$\lim_{x \rightarrow 1} f(x) = -2$$

D

$$\lim_{x \rightarrow 1} f(x) \text{ does not exist}$$

33 For this piecewise function at $x = 2$: does the limit exist, and is the function continuous?



A

The limit exists and the function is continuous

B

The limit does not exist and the function is not continuous

C

The limit exists but the function is not continuous



Rates of Change and Limits

- 34** Is there a hole or a vertical asymptote at $x=1$ when this polynomial is divided by this binomial?

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 3x + 2}$$

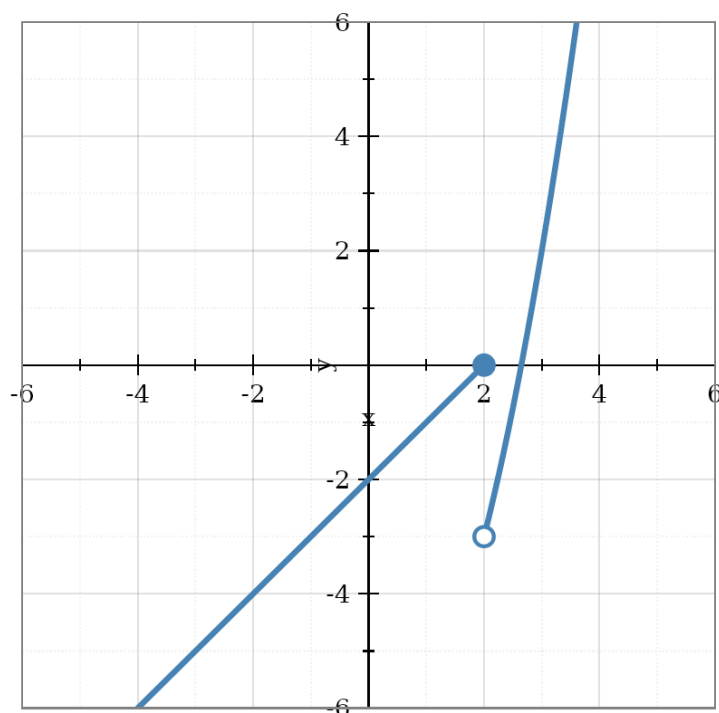
- | | |
|---|--------------------|
| A | Hole |
| B | Vertical Asymptote |
| C | Neither |

- 35** For this piecewise function at $x = -2$: does the limit exist, and is the function continuous?

$$f(x) = \begin{cases} x, & x \leq -2 \\ -x - 4, & x > -2 \end{cases}$$

- | | |
|---|---|
| A | The limit does not exist and the function is not continuous |
| B | The limit exists but the function is not continuous |
| C | The limit exists and the function is continuous |

- 36** Is this function continuous at $x = 2$?



A

Yes

B

No

- 37** Is this function continuous at $x = -2$?

$$f(x) = \begin{cases} 2x + 4, & x < -2 \\ 3, & x = -2 \\ x + 2, & x > -2 \end{cases}$$

- | | | | |
|---|-----|---|----|
| A | Yes | B | No |
|---|-----|---|----|

- 38** Is there a hole or a vertical asymptote at $x=-4$ when this polynomial is divided by this binomial?

$$f(x) = \frac{(x+4)(x+1)}{(x+1)(x-1)}$$

- | | |
|---|--------------------|
| A | Neither |
| B | Vertical Asymptote |
| C | Hole |