



Planes in 3D Space - Intro

- 1 State the normal vector of the plane.

$$4x - 2y + 2z = 4$$

A

$$\begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$$

B

$$\begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$$

C

$$\begin{pmatrix} -4 \\ 2 \\ -2 \end{pmatrix}$$

D

$$\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$$

- 2 Write a possible scalar equation of a plane with the given normal vector.

$$\vec{n} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$$

A

$$3x + 4y = -8$$

B

$$-4x - 3z = -8$$

C

$$5x - y + 3z = -8$$

D

$$4x + 3z = -8$$

- 3 State a point that lies on the plane.

$$\vec{r} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 4 \end{pmatrix}$$

A

$$(0, 6, 1)$$

B

$$(-3, 4, 4)$$

C

$$(3, 2, -3)$$

D

$$(-3, 1, 2)$$

- 4 Which vector is parallel to the plane?

$$\vec{r} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix}$$

A

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

B

$$\begin{pmatrix} 0 \\ -9 \\ 9 \end{pmatrix}$$

C

$$\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$$

D

$$\begin{pmatrix} -3 \\ -6 \\ 12 \end{pmatrix}$$



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5 Write the parametric equations of the plane.

$$\vec{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + s \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} + t \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

A

$$\begin{aligned} x &= 2 + 2s + 4t \\ y &= 1 + -3s + 2t \\ z &= 4 + 3s + 3t \end{aligned}$$

B

$$\begin{aligned} x &= 2 + 2s + 4t \\ y &= -3 + 1s + 2t \\ z &= 3 + 4s + 3t \end{aligned}$$

C

$$\begin{aligned} x &= 2 + 4s + 2t \\ y &= 1 + 2s + -3t \\ z &= 4 + 3s + 3t \end{aligned}$$

6 Write the vector equation of the plane through the point parallel to both vectors.

$$\text{Point } (-4, 1, 2) \quad \vec{u} = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$$

A

$$\vec{r} = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix} + s \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$$

B

$$\vec{r} = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$$

C

$$\vec{r} = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$$



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- 7** Find the scalar equation of the plane through the three points.

A (1, 2, -2) B (0, 2, -2) C (-1, 0, 0)

A $x + z = 0$

B $+y + z = 0$

C $+y + z = 4$

D $-y - z = 0$

- 9** Write the plane in point-normal form.

Point (2, 0, 1) Normal $\begin{pmatrix} -1 \\ -1 \\ -4 \end{pmatrix}$

A $-(x + 2) - (y) - 4(z + 1) = 0$

B $(x - 2) + (y) + 4(z - 1) = 0$

C $-(x - 2) - (y) - 4(z - 1) = 0$

- 8** Find the scalar equation of the plane through the point parallel to both vectors.

Point (1, 3, 0) $\vec{u} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$

A $6x + 3y - 2z = 15$

B $-6x - 3y + 2z = 15$

C $6x + 3y - 2z = 18$

D $-2x + 6y + 3z = 15$

- 10** Find the scalar equation of the plane.

Point (-4, 2, -2) Normal $\begin{pmatrix} 4 \\ -3 \\ -1 \end{pmatrix}$

A $4x - 3y - z = -18$

B $4x - 3y - z = -20$

C $-x + 4y - 3z = -20$

D $-4x + 3y + z = -20$

- 11** Does the given point lie on the plane?

$-4x - 3z = -6$ Point (1, -3, 0)

A No

B Yes

- 12** Find the y-intercept of the plane.

$x + 3y + 2z = 6$

A (0, 2, 0)

B (0, 3, 0)

C (0, 4, 0)

D (0, -2, 0)



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13 Find the vector equation of the normal line to the plane through the given point.

$$+y - z = -6 \quad \text{Point on plane } (2, -2, 4)$$

A

$$\vec{r} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

B

$$\vec{r} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$$

C

$$\vec{r} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$$

D

$$\vec{r} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

14 Find a vector equation of the plane through the three points.

$$\text{A } (4, -3, -3) \quad \text{B } (-4, 3, -1) \quad \text{C } (-1, 1, 4)$$

A

$$\vec{r} = \begin{pmatrix} 4 \\ -3 \\ -3 \end{pmatrix} + s \begin{pmatrix} -8 \\ 6 \\ 2 \end{pmatrix} + t \begin{pmatrix} -5 \\ 4 \\ 7 \end{pmatrix}$$

B

$$\vec{r} = \begin{pmatrix} 4 \\ -3 \\ -3 \end{pmatrix} + s \begin{pmatrix} -4 \\ 3 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix}$$

C

$$\vec{r} = \begin{pmatrix} -8 \\ 6 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ -3 \\ -3 \end{pmatrix} + t \begin{pmatrix} -5 \\ 4 \\ 7 \end{pmatrix}$$



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15 Write a vector equation of the plane through the point parallel to both vectors.

Point $(4, -1, 4)$ $\vec{u} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

A

$$\vec{r} = \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix} + s \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$$

B

$$\vec{r} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$$

C

$$\vec{r} = \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix} + s \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + t \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix}$$

16 Write a vector equation of the plane.

Point $(-1, -2, -3)$ Normal $\begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$

A

$$\vec{r} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} + s \begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix} + t \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$$

B

$$\vec{r} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} + s \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} -9 \\ -18 \\ 9 \end{pmatrix}$$

C

$$\vec{r} = \begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix} + s \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} -9 \\ -18 \\ 9 \end{pmatrix}$$



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17 Does the given point lie on the plane?

$$\vec{r} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} -12 \\ -17 \\ 3 \end{pmatrix} \quad \text{Point } (4, -4, 25)$$

A

No

B

Yes

18 Find the z-intercept of the plane.

$$\vec{r} = \begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix} + s \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} + t \begin{pmatrix} -25 \\ 8 \\ 6 \end{pmatrix}$$

A

(0, 0, -8)

B

(0, 0, 3)

C

(0, 0, 9)

D

(0, 0, 8)

E

(0, 0, 10)

19 Find the vector equation of the normal line to the plane through the given point.

$$\vec{r} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} + s \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -9 \\ 0 \\ 9 \end{pmatrix} \quad \text{Point on plane } (-2, 3, 0)$$

A

$$\vec{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

B

$$\vec{r} = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$$

C

$$\vec{r} = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

D

$$\vec{r} = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

20 The line through the external point perpendicular to the plane meets the plane. Find that point.

$$-2x + y + 4z = 4 \quad \text{External point } (-9, 1, 12)$$

A

(-9, 1, 12)

B

(3, -5, -12)

C

(-3, -2, 0)

D

(-5, -1, 4)



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- 21** Find the vector equation of the normal line to the plane that passes through the external point.

$$-x - y + 3z = 6 \quad \text{External point } (-4, 3, 3)$$

A
$$\vec{r} = \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$$

B
$$\vec{r} = \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix}$$

C
$$\vec{r} = \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

D
$$\vec{r} = \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

E
$$\vec{r} = \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix}$$

- 22** Find the shortest distance from the external point to the plane (to 2 decimal places).
 $3x - 2y - 2z = -6$ External point $(12, -5, -8)$

A 16.49

B 4.00

C 16.98

D 17.37

E 14.84

- 24** Find the scalar equation of the plane containing both lines.

A $20x - 6y + 9z = 45$

B $-20x + 6y - 9z = 45$

C $-9x - 20y + 6z = 45$

D $-20x + 6y - 9z = 46$

- 23** Find the scalar equation of the plane through the point, perpendicular to the given line.

Point $(4, -1, 1)$ Perpendicular line $\vec{r} = \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

A $x - 2y + 3z = -10$

B $-2x + 3y + z = -10$

C $2x - 3y - z = -10$

D $-2x + 3y + z = -9$



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25 Find the scalar equation of the plane containing the point and the line.

Point $(-8, -2, -3)$ Line $\vec{r} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$

A

$$-x = 75$$

B

$$-z = 76$$

C

$$+z = 75$$

D

$$-z = 75$$

28 Select the matrix that highlights the products for the indicated component of the cross product. Green marks the positive product; red marks the negative product.

30 Find the indicated component of the cross product.

A $y = (-1) \cdot (-1) - (-1) \cdot 3 = 4$

B $y = (-1) \cdot 3 + (-1) \cdot (-1) = -2$

C $y = (-1) \cdot 3 - (-1) \cdot (-1) = -4$

D $y = (-1) \cdot 3 - 0 \cdot 3 = -3$

26 Find the scalar equation of the plane through the point, parallel to the given plane.
Point $(1, -3, -2)$ Parallel plane $-x + 4y + 2z = -5$

A

$$x - 4y - 2z = -17$$

B

$$2x - y + 4z = -17$$

C

$$-x + 4y + 2z = -17$$

D

$$-x + 4y + 2z = -13$$

27 Select the matrix that highlights the products for the indicated component of the cross product. Green marks the positive product; red marks the negative product.

29 Find the indicated component of the cross product.

A $x = 1 \cdot (-1) + 1 \cdot (-3) = -4$

B $x = 1 \cdot (-1) - 1 \cdot (-3) = 2$

C $x = 1 \cdot (-4) - (-3) \cdot (-1) = -7$

D $x = 1 \cdot (-3) - 1 \cdot (-1) = -2$

31 Find the indicated component of the cross product.



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32 Find the cross product of the two vectors.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 3 \\ 1 & -4 & 0 \end{vmatrix}$$

A

$$\vec{u} \times \vec{v} = [12, 3, 12]$$

B

$$\vec{u} \times \vec{v} = [3, 12, -12]$$

C

$$\vec{u} \times \vec{v} = [-12, -3, 12]$$

D

$$\vec{u} \times \vec{v} = [12, 3, -12]$$

33 Find the cross product of the two vectors.

$$\vec{u} = [4, 3, -4] \quad \vec{v} = [0, -3, 0]$$

A

$$\vec{u} \times \vec{v} = [-12, 0, 12]$$

B

$$\vec{u} \times \vec{v} = [12, 0, 12]$$

C

$$\vec{u} \times \vec{v} = [-12, 0, -12]$$

D

$$\vec{u} \times \vec{v} = [0, -12, -12]$$

34 Given the normal vectors of two planes, are the planes parallel or coincident?

$$\vec{n}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \vec{n}_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

A

Neither

B

Parallel or coincident

35 What is the relationship between the two planes?

A

Coincident

B

Perpendicular

C

Neither

D

Parallel

36 Are the two planes perpendicular?

A

No

B

Yes

37 Find the acute angle between the two planes.

38 Find the acute angle between the two planes.

$$\vec{n}_1 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \vec{n}_2 = \begin{pmatrix} -1 \\ -4 \\ 1 \end{pmatrix}$$